## Midterm Questions

#### MEGN498A

Due March 19th, 2020 by 2:30pm submitted as a single file (PDF) on CANVAS

Read directions and questions carefully. Only use material provided by or generated from this course, nothing outside or online, as well as a graphing or scientific calculator (or equivalent). Write all steps to get to the final answer out clearly and neatly, box-in final answers and put units (if required) to receive full credit.

Please write out the following phrase, along with your name and signature at the top of your submitted exam to signify you followed the rules. Failure to do will result in a 15 point penalty:

I followed the Mines Honor Code on this exam. [- Printed Name & Signature]

Question 1: (10 points)

The components of a displacement field for a structure are given by (in mm):

$$u = x^3 + 20$$

$$v = y^2$$

$$w=z^3$$

Showing work to justify answers:

- (A) Determine all 6 components of strain.
- (B) Is this an admissible solution? Show full work to receive credit, as well as answer YES or NO.

Question 2: (4 points)

If you are given a displacement field of a structure and you want to find the corresponding stress field, please state (in the correct order) the order of the 5 ingredients you would use to solve such a problem.

Question 3: (7 points)

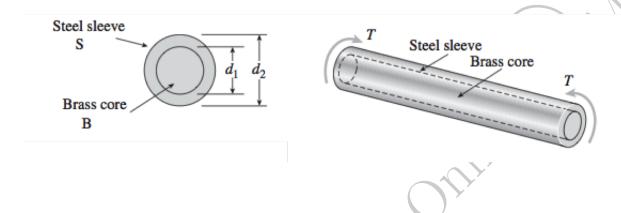
A Saturn V rocket 3,500,000 kg in total weight, 80% of which is propellant fuel, rises vertically by ejecting exhaust thrust gases at a constant velocity of 6.3 km/sec and consumes propellant fuel at a constant rate of 8,000 kg/sec.

- (A) How many seconds can the rocket burn before the propellant is totally consumed at final burnout?
- (B) Calculate the burnout velocity of the rocket.

### Question 4: (7 points)

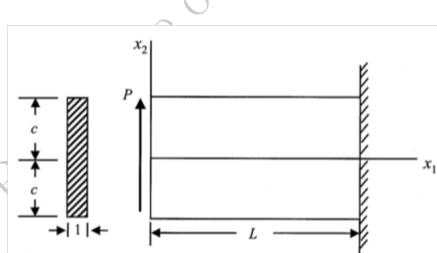
A composite helicopter shaft (made out of 2 isotropic materials) is manufactured by shrink-fitting a steel outer sleeve over a brass core. This allows the two parts to act as a single solid rod in torsion. The outer diameters are  $d_1 = 40$  mm for the brass core, and  $d_2 = 50$  mm for the steel sleeve. The shear moduli of elasticity are  $G_b = 38$  GPa for the brass, and  $G_s = 80$  GPa for the steel.

The allowable shear stresses in the brass and steel are  $\tau_b = 50$  MPa and  $\tau_s = 80$  MPa, respectively. Determine the maximum permissible torque,  $T_{max}$ , that may be applied to the shaft.



## Question 5: (12 points)

Given the following Airy's stress function for a cantilever beam with unit depth (rectangular cross-section) with a shear loading on the free end such that the average shear stress is equivalent to the vertical load, P. Note that  $x_1 = x$  and  $x_2 = y$  in the image and  $x_1 = 0$  is at the free end of the beam and  $x_1 = L$  is at the fixed end, and the top and bottom of the beam are  $y = \pm c$ , respectively.



# $\phi(x,y) = Dxy^3 + Bxy$

## Determine:

- (A) Does this stress function satisfy equilibrium and compatibility? Show your work to demonstrated yes or no and be sure to state your final answer.
- (B) Assuming it does satisfy the above, determine the values of B and D.
- (C) Determine the non-zero stress distribution in the beam  $(\sigma_x, \sigma_y, \sigma_{xy})$ .